Claims

[c1]   1. A method for modeling a characteristic C that is distributed within a domain, said method comprising:

   providing a base equation expressing the characteristic C as a function f of a variable V through use of $N+1$ parameters C_0, C_1, ..., C_N, said base equation being of the form $C = f(C_0, C_1, ..., C_N, V)$, said N being at least 1, said parameters C_0, C_1, ..., C_N being subject to uncertainty;

   providing a probability density function (PDF) for describing the probability of occurrence of C_0 in accordance with said uncertainty; and

   providing subsidiary equations expressing C_1, ..., C_N in terms of C_0.

[c2]   2. The method of claim 1, wherein $C = C_0$ if $V = V_0$, and wherein providing the PDF comprises:

   providing test data of $C = C_0(k)$ at each node k of K nodes in a space such that $V = V_0$ at node k, said $C_0(k)$ being C_0 at node k, said K being at least 2, said k being an integer having values 1, 2, ..., K; and

   deriving said PDF from said test data.

[c3]   3. The method of claim 1, wherein providing the sub-

sidiary equations comprise:

provising test data of $C(k)$ versus V at each node k of K nodes in a space, said $C(k)$ being C at node k, said K being at least 2, said k being an integer having values 1, 2, ..., K;

fitting the function f to the test data at each node k of the K nodes to obtain $C(k) = f(C_0(k), C_1(k), ..., C_N(k), V)$, said $C_0(k), C_1(k), ..., C_N(k)$ respectively denoting $C_0, C_1, ..., C_N$ at node k; and

deriving the subsidiary equations by utilizing $C(k) = f(C_0(k), C_1(k), ..., C_N(k), V)$ at each node k of the K nodes.

[c4]   4. The method of claim 1, further comprising:

providing a value V'' of V;

picking a random value C_{0R} of C_0 from the PDF;

computing values $C_{1R}, ..., C_{NR}$ of $C_1, ..., C_N$, respectively, by substituting C_{0R} into the subsidiary equations; and

calculating a value of C by substituting $C_{0R}, C_{1R}, ..., C_{NR}$ and V'' into the base equation.

[c5]   5. The method of claim 1, further comprising determining a performance characteristic of a design, said design comprising I nodes in the domain, said I being at least 2, each node i of the I nodes having a value $C(i)$ of the characteristic C, said i having values of 1, 2, ..., I, said

determining a performance characteristic comprising:

> randomly selecting a value of C(i) of C at each node i of the I nodes; and
>
> determining the performance characteristic, including utilizing said randomly selected C(1), C(2), ..., C(I).

[c6] 6. The method of claim 5, said randomly selecting a value of C(i) comprising:

> providing a value V(i) of V at node i;
>
> picking a random value C_{0R} of C_0 from the PDF;
>
> calculating corresponding values C_{1R}, ..., C_{NR} of C_1, ..., C_N, respectively, by substituting C_{0R} into the subsidiary equations; and
>
> computing C(i) by substituting C_{0R}, C_{1R}, ..., C_{NR} and V(i) into the base equation.

[c7] 7. The method of claim 1, said PDF being a normal probability distribution.

[c8] 8. The method of claim 1, said subsidiary equations having a form of $C_x = g_x(C_{x-1})$ for functions g_x, said functions g_x each being a linear of quadratic function of C_{x-1}, said x having values of 1, 2, ..., N.

[c9] 9. The method of claim 1, said domain being a physical domain, said characteristic C being spatially distributed

within said physical domain.

[c10] 10. The method of claim 1, said characteristic C being an electrical characteristic.

[c11] 11. The method of claim 10, said characteristic C denoting capacitance at a node of the domain, said V denoting a voltage applied to the node.

[c12] 12. The method of claim 11, said function f being a polynomial in V of order N, said N being at least 5.

[c13] 13. The method of claim 12, said N=2, said function f being $C_0/(1-V/V_B)^m$, said $C_1=V_B$, said $C_2=m$.

[c14] 14. A method for modeling a characteristic C that is distributed within a domain, said characteristic C having J subcharacteristics S_1, S_2, ..., S_J, said method comprising:

provided a combination equation that expresses C as a function F of the J subcharacteristics, said J being at least 2;

providing base equations expressing S_j as a function f_j of a variable V through use of N+1 parameters S_{j0}, S_{j1}, ..., S_{jN}, said base equations being of the form $S_j = f_j(S_{j0}, S_{j1}, ..., S_{jN}, V)$, said N being at least 1, said parameters S_{j0}, S_{j1}, ..., S_{jN} being subject to uncertainty, said j having values of 1, 2, ..., J;

providing at least one probability density function

(PDF) from PDF_1, PDF_2, ..., PDF_J, said PDF_n describing the probability of occurrence of S_{n0} in accordance with said uncertainty for n=1, 2, ..., J, said at least one PDF including PDF_1;

for each PDF_n not provided: providing an auxiliary equation E_n expressing S_{n0} in terms of S_{10}; and providing subsidiary equations expressing S_{j1}, ..., S_{jN} in terms of S_{j0} for each subcharacteristic S_j of the J subcharacteristics.

[c15]   15. The method of claim 14, wherein $S_j = S_{j0}$ if $V = V_0$ for each subcharacteristic S_j of the J subcharacteristics, and wherein providing the PDF_n for the at least one n comprises:

   providing test data from which $S_{n0}(k)$ may be inferred at each node k of K nodes in a space such that $V = V_0$ at node k, said $S_{n0}(k)$ being S_{n0} at node k, said K be-ing at least 2, said k having values of 1, 2, ..., K; and deriving said PDF_n from said test data.

[c16]   16. The method of claim 14, wherein providing the sub-sidiary equations comprise for each subcharacteristic S_j of the J subcharacteristics:

   providing test data from which $S_j(k)$ versus V may be inferred at each node k of K nodes in a space, said $S_j(k)$ being S_j at node k, said K being at least 2, said k having values of 1, 2, ..., K;

fitting the function f_j to the test data at each node k of the K nodes to obtain $S_j(k) = f(S_{j0}(k), S_{j1}(k), ..., S_{jN}(k), V)$, said $S_{j0}(k), S_{j1}(k), ..., S_{jN}(k)$ respectively denoting $S_{j0}, S_{j1}, ..., S_{jN}$ at each node k; and

deriving the subsidiary equations by utilizing $S_j(k) = f(S_{j0}(k), S_{j1}(k), ..., S_{jN}(k), V)$ for each node k of the K nodes.

[c17]   17. The method of claim 14, further comprising:

providing a value V" of V;

for each PDF_n provided: picking a random value of S_{n0R} from PDF_n;

for PDF_n not provided: calculating S_{n0R} by substituting S_{10R} into the auxiliary equation E_n;

computing values $S_{j1R}, ..., S_{jNR}$ of $S_{j1}, ..., S_{jN}$, respectively, by substituting S_{j0R} into the subsidiary equations for each subcharacteristic S_j of the J subcharacteristics;

calculating S_j by substituting $S_{j0R}, S_{j1R}, ..., S_{jNR}$ and V" into the base equations $S_j = f_j(S_{j0}, S_{j1}, ..., S_{jN}, V)$ for each subcharacteristic S_j of the J subcharacteristics; and

computing C by substituting $S_1, S_2, ..., S_J$ into the combination equation.

[c18]   18. The method of claim 14, further comprising determining a performance characteristic of a design, said

design comprising I nodes in the domain, said I being at least 2, each node of the I nodes having the characteristic C comprising the J subcharacteristics S_1, S_2, ..., S_J, said I having values of 1, 2, ..., I, said determining a performance characteristic comprising:

randomly selecting a value C(i) of C at each node i of the I nodes, including:

randomly selecting a value $S_j(i)$ of S_j at node i for each subcharacteristic S_j of the J subcharacteristics

and computing C(i) by substituting $S_1(i)$, $S_2(i)$, ..., $S_J(i)$ into the combination equation for each node i; and

determining said performance characteristic, including utilizing said randomly selected C(1), C(2), ..., C(I).

[c19]   19. The method of claim 18, said randomly selecting $S_j(i)$ at node i for each subcharacteristic S_j of the J subcharacteristics comprising:

providing a value V(i) of V at node i;

for each n such that PDF_n is provided for: picking a random value of S_{nOR} from PDF_j;

for each n such that PDF_n is not provided for: calculating S_{nOR} by substituting S_{1OR} into the auxiliary equation E_n;

calculating corresponding values S_{j1R}, ..., S_{jNR} of S_{j1}, ..., S_{jN}, respectively, by substituting S_{jOR} into the sub-

sidiary equations; and

calculating $S_j(i)$ by substituting S_{j0R}, S_{j1R}, ..., S_{jNR} and $V(i)$ into the base equation of S_j.

[c20] 20. The method of claim 14, said method further comprising for $n=2$, ..., J: determining whether S_{n0} is sufficiently correlated with S_{10}, wherein if said determining determines that S_{n0} is not sufficiently correlated with S_{10} then providing said PDF_n, but if said determining determines that S_{n0} is sufficiently correlated with S_{10} then not providing said PDF_n and instead deriving said auxiliary equation E_n from scatter data of S_{n0} versus S_{10}.

[c21] 21. The method of claim 20, wherein said determining whether S_{n0} is sufficiently correlated with S_{10} comprises computing a correlation parameter R_n from said scatter data of S_{n0} versus S_{10}, said correlation parameter R_n being a square of a correlation coefficient r_n between S_{n0} and S_{10}, and wherein S_{no} is sufficiently correlated with S_{10} if R_n is no less than a specified minimum correlation parameter R_{MIN}.

[c22] 22. The method of claim 14, said f_j having a same functional form of V for each subcharacteristic S_j of the J subcharacteristics.

[c23] 23. The method of claim 14, said f_j being constant with

respect to any variation in V for each subcharacteristic S_j of the J subcharacteristics.

[c24] 24. The method of claim 14, said f_j varying with respect to a variation in V for each subcharacteristic S_j of the J subcharacteristics.

[c25] 25. The method of claim 14, said PDF_j being a normal probability distribution for at least one subcharacteristic S_j of the J subcharacteristics.

[c26] 26. The method of claim 14, said domain being a physical domain, said characteristic C being spatially distributed within said physical domain.

[c27] 27. The method of claim 14, said characteristic C being an electrical characteristic.

[c28] 28. The method of claim 27, said characteristic C denoting capacitance at a node of the domain, said V denoting a voltage applied to the node.

[c29] 29. The method of claim 28, said function f_j being a polynomial in V of order N for each subcharacteristic S_j of the J subcharacteristics, said N being at least 5.

[c30] 30. A computer program product, comprising a computer usable medium having a computer readable program code embodied therein, said computer readable

program code adapted to be executed on a processor for implementing a method for modeling a characteristic C that is distributed within a domain, said method comprising:

   providing a base equation expressing the characteristic C as a function f of a variable V through use of $N+1$ parameters C_0, C_1, ..., C_N, said base equation being of the form $C = f(C_0, C_1, ..., C_N, V)$, said N being at least 1, said parameters C_0, C_1, ..., C_N being subject to uncertainty;

   providing a probability density function (PDF) for describing the probability of occurrence of C_0 in accordance with said uncertainty; and

   providing subsidiary equations expressing C_1, ..., C_N in terms of C_0.

[c31]   31. The computer program product of claim 30, further comprising:

   providing a value V" of V;

   picking a random value C_{OR} of C_0 from the PDF;

   computing values C_{1R}, ..., C_{NR} of C_1, ..., C_N, respectively, by substituting C_{OR} into the subsidiary equations; and

   calculating a value of C by substituting C_{OR}, C_{1R}, ..., C_{NR} and V" into the base equation.

[c32] 32. The computer program product of claim 30, further comprising determining a performance characteristic of a design, said design comprising I nodes in the domain, said I being at least 2, each node i of the I nodes having a value $C(i)$ of the characteristic C, said i having values of 1, 2, ..., I, said determining a performance characteristic comprising:

randomly selecting a value of $C(i)$ of C at each node i of the I nodes; and

determining the performance characteristic, including utilizing said randomly selected $C(1)$, $C(2)$, ..., $C(I)$.

[c33] 33. The computer program product of claim 32, said randomly selecting a value of $C(i)$ comprising:

providing a value $V(i)$ of V at node i;

picking a random value C_{OR} of C_0 from the PDF;

calculating corresponding values C_{1R}, ..., C_{NR} of C_1, ..., C_N, respectively, by substituting C_{OR} into the subsidiary equations; and

computing $C(i)$ by substituting C_{OR}, C_{1R}, ..., C_{NR} and $V(i)$ into the base equation.

[c34] 34. A computer program product, comprising a computer usable medium having a computer readable program code embodied therein, said computer readable program code adapted to be executed on a processor for

implementing a method for modeling a characteristic C that is distributed within a domain, said characteristic C having J subcharacteristics S_1, S_2, ..., S_J, said method comprising:

providing a combination equation that expresses C as a function F of the J subcharacteristics, said J being at least 2;

providing base equations expressing S_j as a function f_j of a variable V through use of N+1 parameters S_{j0}, S_{j1}, ..., S_{jN}, said base equations being of the form $S_j = f_j(S_{j0}, S_{j1}, ..., S_{jN}, V)$, said N being at least 1, said parameters S_{j0}, S_{j1}, ..., S_{jN} being subject to uncertainty, said j having values of 1, 2, ..., J;

providing at least one probability density function (PDF) from PDF_1, PDF_2, ..., PDF_J, said PDF_n describing the probability of occurrence of S_{n0} in accordance with said uncertainty for n=1, 2, ..., J, said at least one PDF including PDF_1;

for each PDF_n not provided: providing an auxiliary equation E_n expressing S_{n0} in terms of S_{10}; and providing subsidiary equations expressing S_{j1}, ..., S_{jN} in terms of S_{j0} for each subcharacteristic S_j of the J subcharacteristics.

[c35] 35. The computer program product of claim 34, further comprising:

providing a value V'' of V;

for each PDF_n provided: picking a random value of S_{nOR} from PDF_n;

for PDF_n not provided: calculating S_{nOR} by substituting S_{10R} into the auxiliary equation E_n;

computing values $S_{j1R}, ..., S_{jNR}$ of $S_{j1}, ..., S_{jN}$, respectively, by substituting S_{jOR} into the subsidiary equations for each subcharacteristic S_j of the J subcharacteristics;

calculating S_j by substituting $S_{jOR}, S_{j1R}, ..., S_{jNR}$ and V'' into the base equations $S_j = f_j(S_{j0}, S_{j1}, ..., S_{jN}, V)$ for each subcharacteristic S_j of the J subcharacteristics; and

computing C by substituting $S_1, S_2, ..., S_J$ into the combination equation.

[c36] 36. The computer program product of claim 34, further comprising determining a performance characteristic of a design, said design comprising I nodes in the domain, said I being at least 2, each node of the I nodes having the characteristic C comprising the J subcharacteristics $S_1, S_2, ..., S_J$, said I having values of 1, 2, ..., I, said determining a performance characteristic comprising:

randomly selecting a value $C(i)$ of C at each node i of the I nodes, including:

randomly selecting a value $S_j(i)$ of S_j at node i for

each subcharacteristic S_j of the J subcharacteristics and computing C(i) by substituting $S_1(i, S_2(i, ..., S_j(i$ into the combination equation for each node i and determining said performance characteristic, including utilizing said randomly selected C(1), C(2), ..., C(I).

[c37] 37. The computer program product of claim 36, said randomly selecting $S_j(i)$ at node i for each subcharacteristic S_j of the J subcharacteristics comprising:

provid ing a value V(i) of V at node i;

for each n such that PDF_n is provided for: picking a random value of S_{nOR} from PDF_j;

for each n such that PDF_n is not provided for: calculating S_{nOR} by substituting S_{1OR} into the auxiliary equation E_n;

calculating corresponding values S_{j1R}, ..., S_{jNR} of S_{j1}, ..., S_{jN}, respectively, by substituting S_{jOR} into the subsidiary equations; and

calculating $S_j(i)$ by substituting S_{jOR}, S_{j1R}, ..., S_{jNR} and V(i) into the base equation of S_j.

[c38] 38. The computer program product of claim 34, said method further comprising for n=2, ..., J: determining whether S_{n0} is sufficiently correlated with S_{10}, wherein if said determining determines that S_{n0} is not sufficiently correlated with S_{10} then providing said PDF_n, but if said

determining determines that S_{n0} is sufficiently correlated with S_{10} then not providing said PDF_n and instead deriving said auxiliary equation E_n from scatter data of S_{n0} versus S_{10}.

[c39]   39. The computer program product of claim 38, wherein said determining whether S_{n0} is sufficiently correlated with S_{10} comprises computing a correlation parameter R_n from said scatter data of S_{n0} versus S_{10}, said correlation parameter R_n being a square of a correlation coefficient r_n between S_{n0} and S_{10}, and wherein S_{no} is sufficiently correlated with S_{10} if R_n is no less than a specified minimum correlation parameter R_{MIN}.

[c40]   40. A model, comprising:

a base equation expressing a characteristic C as a function f of a variable V through use of N+1 parameters C_0, C_1, ..., C_N, said base equation being of the form $C = f(C_0, C_1, ..., C_N, V)$, said N being at least 1, said parameters C_0, C_1, ..., C_N being subject to uncertainty, said characteristic C being distributed within a domain;

a probability density function (PDF) for describing the probability of occurrence of C_0 in accordance with said uncertainty; and

subsidiary equations expressing C_1, ..., C_N in terms of C_0.

[c41] 41. The model of claim 40, further comprising means for using the base equation, the PDF, and the subsidiary equations to calculate a value of C from input comprising a value V" of V.

[c42] 42. The model of claim 40, further comprising means for using the base equation, the PDF, and the subsidiary equations to determine a performance characteristic of a design, said design comprising I nodes in the domain, said i being at least 2, each node i of the I nodes having a value C(i) of the characteristic C, said i having values of 1, 2, ..., I.

[c43] 43. The model of claim 40, said PDF being a normal probability distribution.

[c44] 44. The model of claim 40, said subsidiary equations having a form of $C_x = g_x(C_{x-1})$ for functions g_x, said functions g_x each being a linear of quadratic function of C_{x-1}, said x having values of 1, 2, ..., N.

[c45] 45. The model of claim 40, said domain being a physical domain, said characteristic C being spatially distributed within said physical domain.

[c46] 46. The model of claim 45, said characteristic C being an electrical characteristic.

[c47] 47. The model of claim 46, said characteristic C denoting capacitance at a node of the domain, said V denoting a voltage applied to the node.

[c48] 48. The model of claim 47, said function f being a polynomial in V of order N, said N being at least 5.

[c49] 49. A model, comprising:

a combination equation that expresses C as a function F of the J subcharacteristics, said J being at least 2, said characteristic C being distributed within a domain, said characteristic C having J subcharacteristics $S_1, S_2, ..., S_J$;

base equations expressing S_j as a function f_j of a variable V through use of N+1 parameters $S_{j0}, S_{j1}, ...,$ S_{jN}, said base equations being of the form $S_j = f_j(S_{j0},$ $S_{j1}, ..., S_{jN}, V)$, said N being at least 1, said parameters $S_{j0}, S_{j1}, ..., S_{jN}$ being subject to uncertainty, said j having values of 1, 2, ..., J;

at least one probability density function (PDF) from $PDF_1, PDF_2, ..., PDF_J$, said PDF_n describing the probability of occurrence of S_{n0} in accordance with said uncertainty for n=1, 2, ..., J, said at least one PDF including PDF_1;

for each PDF_n that does not exist: an auxiliary equation E_n expressing S_{n0} in terms of S_{10}; and

subsidiary equations expressing $S_{j1}, ..., S_{jN}$ in terms of S_{j0} for each subcharacteristic S_j of the J subcharacteristics.

[c50]   50. The model of claim 49, further comprising means for using the combination equation, the base equations, the at least one PDF, each auxiliary equation, and the subsidiary equations to calculate a value of C from input comprising a value V" of V.

[c51]   51. The model of claim 49, further comprising means for using the combination equation, the base equations, the at least one PDF, each auxiliary equation, and the subsidiary equations to

determine a performance characteristic of a design, said design comprising I nodes in the domain, said I being at least 2, each node i of the I nodes having the characteristic C comprising the J subcharacteristics $S_1, S_2, ..., S_j$, said i having values of 1, 2, ..., I.

[c52]   52. The model of claim 49, said f_j having a same functional form of V for each subcharacteristic S_j of the J subcharacteristics.

[c53]   53. The model of claim 49, said f_j being constant with respect to any variation in V for each subcharacteristic S_j of the J subcharacteristics.

[c54]   54. The model of claim 49, said f_j varying with respect to a variation in V for each subcharacteristic S_j of the J subcharacteristics.

[c55]   55. The model of claim 49, said PDF_j being a normal probability distribution for at least one subcharacteristic S_j of the J subcharacteristics.

[c56]   56. The model of claim 49, said domain being a physical domain, said characteristic C being spatially distributed within said physical domain.

[c57]   57. The model of claim 56, said characteristic C being an electrical characteristic.

[c58]   58. The model of claim 57, said characteristic C denoting capacitance at a node of the domain, said V denoting a voltage applied to the node.

[c59]   59. The model of claim 58, said function f_j being a polynomial in V of order N for each subcharacteristic S_j of the J subcharacteristics, said N being at least 5.